SADLER MATHEMATICS METHODS UNIT 2

WORKED SOLUTIONS

Chapter 6 Applications of differentiation

Exercise 6A

Question 1

$$\frac{dQ}{dr} = 10r + 3$$

Question 2

$$\frac{dX}{dk} = 3 + 6k - 18k^2$$

Question 3

$$\frac{dT}{dr} = 15r^2 - 2r + 15$$

Question 4

$$\frac{dQ}{dp} = 8p^3 + 9p^2 - 14$$

Question 5

$P = 12t^3 + 9t^2 - 8t - 6$

$\frac{dP}{dt} = 36t^2 + 18t - 8$

Question 6

$\frac{dA}{dt} = 10t + 6$

a $\frac{dA}{dt} = 10(1) + 6 = 16$

b $\frac{dA}{dt} = 10(2) + 6 = 26$

c $\frac{dA}{dt} = 10(3) + 6 = 36$

Question 7

$\frac{dP}{da} = 6a$

a $\frac{dP}{da} = 6(2) = 12$

b $\frac{dP}{da} = 6(3) = 18$

c $\frac{dP}{da} = 6(-4) = -24$

Question 8

$$\frac{dA}{dr} = 2\pi r$$

a $\frac{dA}{dr} = 2\pi(10) = 20\pi$

b $\frac{dA}{dr} = 2\pi(3) = 6\pi$

c $\frac{dA}{dr} = 2\pi(\frac{70}{\pi}) = 140$

Question 9

$$\frac{dA}{dr} = 4\pi r + 20\pi$$

a $\frac{dA}{dr} = 4\pi(3) + 20\pi = 32\pi$

b $\frac{dA}{dr} = 4\pi(7) + 20\pi = 48\pi$

c $\frac{dA}{dr} = 4\pi(10) + 20\pi = 60\pi$

Question 10

$$\frac{dV}{dr} = 4\pi r^2$$

a $\frac{dV}{dr} = 4\pi(1)^2 = 4\pi$

b $\frac{dV}{dr} = 4\pi(3)^2 = 36\pi$

c $\frac{dV}{dr} = 4\pi(10)^2 = 400\pi$

Question 11

a $A = \pi r^2$

$= \pi\left(\frac{2t}{5}\right)^2$

$= \frac{4\pi t^2}{25}$

b $A = \frac{4\pi(2)^2}{25}$

$= \frac{16\pi}{25}$

c $\frac{dA}{dtx} = \frac{8\pi t}{25}$

d $\frac{dA}{dtx} = \frac{8\pi(3)}{25}$

$= \frac{24\pi}{25}$

Question 12

a $N = 120 + 5000(0) + 10(0)^3$

$= 120$

b $N = 120 + 5000(5) + 10(5)^3$

$= 3870$

c $\frac{3870 - 120}{5} = 750$ bacteria/h

d $\frac{dN}{dt} = 500 + 30t^2$

e **i** $\frac{dN}{dt} = 500 + 30(2)^2 = 620$ bacteria/h

ii $\frac{dN}{dt} = 500 + 30(5)^2 = 1250$ bacteria/h

iii $\frac{dN}{dt} = 500 + 30(10)^2 = 3500$ bacteria/h

Question 13

a $n = 42(8) + 9(8)^2 - 8^3$

$= 400$ units

b $\frac{400}{8} = 50$ units/h

c $n = 42(7) + 9(7)^2 - 7^3$

$= 392$ units

In the 8th hour, 400 – 392 = 8 units were produced

d $\frac{dN}{dt} = 42 + 18t - 3t^2$

i $\frac{dN}{dt} = 42 + 18(1) - 3(2)^2$

$= 57$ units/h

ii $\frac{dN}{dt} = 42 + 18(2) - 3(2)^2$

$= 66$ units/h

iii $\frac{dN}{dt} = 42 + 18(3) - 3(3)^2$

$= 69$ units/h

Question 14

a **i** $V = \frac{10}{1000}(10 + 10) = 0.2 \text{ L}$

ii $t = 24 \times 60 = 1440$

$V = \frac{1440}{1000}(1440 + 10) = 2088 \text{ L}$

b $V = \frac{t^2}{1000} + \frac{10t}{1000}$

$\frac{dV}{dt} = \frac{t}{500} + 0.01$

i $\frac{dV}{dt} = \frac{10}{500} + 0.01 = 0.03 \text{ L / min}$

ii $\frac{dV}{dt} = \frac{120}{500} + 0.01 = 0.25 \text{ L / min}$

iii $\frac{dV}{dt} = \frac{1440}{500} + 0.01 = 2.89 \text{ L / min}$

Question 15

a **i** $P = 40 + \frac{1(1+20)}{10} = 42.1$

42 deer

ii $P = 40 + \frac{2(2+20)}{10} = 44.1$

44 deer

iii $P = 40 + \frac{3(3+20)}{10} = 46.9$

47 deer

iv $P = 40 + \frac{10(10+20)}{10} = 70$

70 deer

b $\frac{dP}{dt} = \left(\frac{t}{5} + 2\right)$ deer/year

c **i** $\frac{dP}{dt} = \frac{5}{5} + 2 = 3$ deer/year

ii $\frac{dP}{dt} = \frac{10}{5} + 2 = 4$ deer/year

iii $\frac{dP}{dt} = \frac{20}{5} + 2 = 6$ deer/year

Question 16

a $T = 20(0)^3 - 420(0)^2 - 8000(0) - 150000$

$= 150\,000$ tonnes

b $T = 20(10)^3 - 420(10)^2 - 8000(10) - 150000$

$= 48\,000$ tonnes

c $\dfrac{dT}{dt} = 60t^2 - 840t - 8000$

The rate of change is negative, so the rate of increase is

$-(60t^2 - 840t - 8000) = 8000 + 840t - 60t^2$

d **i** $\dfrac{dT}{dt} = 8000 + 840(2) - 60(2)^2$

$= 9440$

decreasing at 9440 tonnes/year

ii $\dfrac{dT}{dt} = 8000 + 840(4)^2 - 60(4)^2$

$= 10\,400$

decreasing at 10 440 tonnes/year

iii $\dfrac{dT}{dt} = 8000 + 840(7) - 60(7)^2$

$= 10\,940$

decreasing at 10 940 tonnes/year

Question 17

a $V = 1000 - 4(0) + \frac{1}{10}(0)^2$

$= 1000 \text{ cm}^3$

b $V = 1000 - 4(2) + \frac{1}{10}(2^2$

$= 992.4 \text{ cm}^3$

c $\frac{dV}{dt} = -4 + \frac{t}{5}$

d **i** $\frac{dV}{dt} = -4 + \frac{0}{5} = -4$

decreasing by 4 cm^3/s

ii $\frac{dV}{dt} = -4 + \frac{3}{5} = -3.4$

decreasing by 3.4 cm^3/s

e Puncture is repaired when the volume stops decreasing

$\frac{dV}{dt} = -4 + \frac{t}{5} = 0$

$\frac{t}{5} = 4$

$t = 20$

Compound takes 20 seconds to repair puncture

f At the time of puncture, $t = 0 \Rightarrow a = 0$

The puncture is repaired at $t = 20 \Rightarrow b = 20$

Exercise 6B

Question 1

a $$\frac{dy}{dx}=3x^2+6x-45=0$$
$$3(x^2-2x-15)=0$$
$$3(x-5)(x+3)=0$$
$$x=-3, x=5$$

The curve has two stationary points at $x=-3, x=5$

at $x=5$

$y=5^3+3(5)^2+45(5)=-195$

$\Rightarrow (5,-195)$ is a local minimum

b At a turning point of a curve, the derivative is zero.

For this curve, there are only two places at which the derivative is zero, as shown in **a**.

Both of these are shown on the calculator display.

c at $x=-3$

$y=(-3)^3+3(-3)^2-45(-3)-20$

$=61$

$\Rightarrow (-3,\ 61)$

Question 2

a $\frac{dy}{dx} = 3x^2 + 3x - 36 = 0$

$3(x^2 + x - 12) = 0$

$3(x+4)(x-3) = 0$

$x = -4, x = 3$

at $x = -4$

$y = (-4)^3 + 1.5(-4)^2 - 36(-4) + 17 = 121$

$\Rightarrow (-4, 121)$ is a local maximum

b At a turning point of a curve, the derivative is zero.

For this curve, there are only two places at which the derivative is zero, as shown in **a**.

Both of these are shown on the calculator display.

c at $x = 3$

$y = (3)^3 + 1.5(3)^2 - 36(3) + 17 = -50.5$

$\Rightarrow (3, -50.5)$ is a local minimum

Question 3

$$\frac{dy}{dx} = 3x^2 + 6x - 9 = 0$$
$$3(x^2 + 2x - 3) = 0$$
$$3(x-1)(x+3) = 0$$
$$\Rightarrow x = -3, x = 1$$

When $x = -3$,
$y = (-3)^3 + 3(-3)^2 - 9(-3) - 7 = 20$
When $x = 1$,
$y = (1)^3 + 3(1)^2 - 9(1) - 7 = -12$

Stationary points exist at (–3, 20) and (1, –12)

Investigation of the sign of the derivative on either side will indicate the type of stationary point.

x	–3.1	–3	–2.9
$\frac{dy}{dx}$	1.23	0	–1.17
	/	—	\

There is a maximum turning point at $x = -3$.

Exact calculation of the value of the derivative is not necessary, but is easily done in Classpad.

Main, Interactive, Calculation, diff produce the following

```
diff(x^3+3·x^2-9·x-7,x,1,-3.1)
                          1.23
```

We really need only to investigate the sign of the derivative.

For future examples the process is shown.

When $x=-3.1$, and using the factorised version of the derivative,

$$\frac{dy}{dx}=3(-3.1+3)\times(-3.1-1)$$

The sign of $\frac{dy}{dx}$ is positive as we have a positive number multiplied by two negative numbers.

Similarly, when $x=-2.9$

$$\frac{dy}{dx}=3(-2.9+3)\times(-2.9-1)$$

The sign of $\frac{dy}{dx}$ is negative as we have a two positive number multiplied by a negative number.

Using this process to investigate the stationary point at $x=1$,

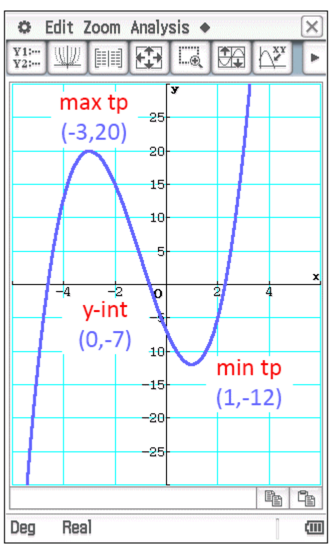
x	0.9	1	1.1
$\frac{dy}{dx}$	$-$ve	0	$+$ve
	\	—	/

There is a minimum turning point at $x=1$.

As $x\to\infty$, the x^3 term in the function dominates so $y\to\infty$.

Similarly, as $x\to-\infty$, $y\to-\infty$.

The curve has a y-intercept of (0, –7.)



Question 4

$$\frac{dy}{dx} = 3x^2 - 18x + 15 = 0$$

$$3(x^2 - 6x + 5) = 0$$

$$3(x-1)(x-5) = 0$$

$$\Rightarrow x = 1, x = 5$$

When $x = 1$,

$y = (1)^3 - 9(1)^2 + 15(1) + 30 = 37$

When $x = 5$,

$y = (5)^3 - 9(5)^2 + 15(5) + 30 = 5$

Stationary points exist at (1, 37) and (5, 5).

Investigation of the sign of the derivative on either side will indicate the type of stationary point.

x	0.9	1	1.1
$\frac{dy}{dx}$	+ve	0	−ve
	/	—	\

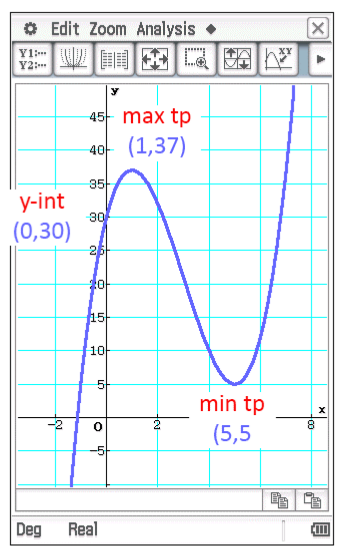
There is a maximum turning point at $x = 1$.

x	4.9	5	5.1
$\frac{dy}{dx}$	−ve	0	+ve
	\	—	/

There is a minimum turning point at $x = 5$.

As $x \to \infty$, the x^3 term in the function dominates so $y \to \infty$.
Similarly, as $x \to -\infty$, $y \to -\infty$.

The curve has a y-intercept of (0, 30).



Question 5

$$\frac{dy}{dx} = -4x + 8 = 0$$

$$\Rightarrow x = 2$$

When $x = 2$,

$y = 1 + 8(2) - 2(2)^2 = 9$

A stationary point exists at (2, 9).

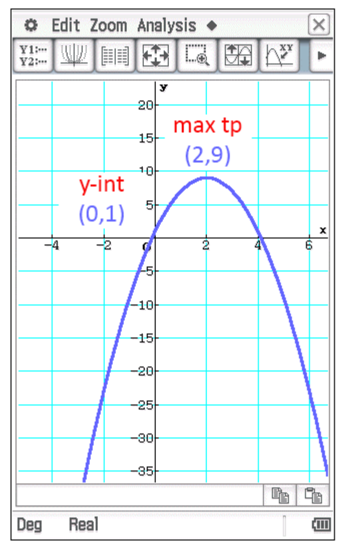
Investigation of the sign of the derivative on either side will indicate the type of stationary point.

x	1.9	2	2.1
$\frac{dy}{dx}$	+ve	0	−ve
	/	—	\

There is a maximum turning point at $x = 2$.

As $x \to \infty$, the $-2x^2$ term in the function dominates so $y \to -\infty$.
Similarly, as $x \to -\infty$, $y \to -\infty$.

The curve has a y-intercept of (0, 1).



Question 6

$$\frac{dy}{dx} = 5x^4 = 0$$
$$\Rightarrow x = 0$$

When $x = 0$,
$y = (0)^5 = 0$

A stationary point exists at (0, 0).

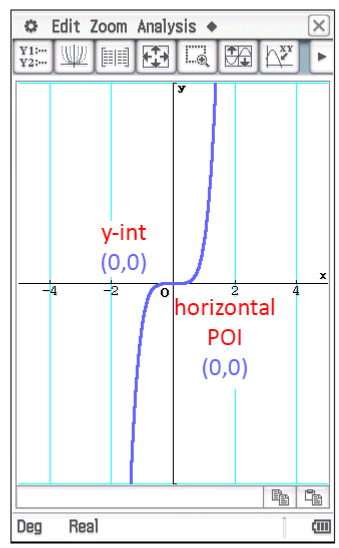
Investigation of the sign of the derivative on either side will indicate the type of stationary point.

x	–0.1	0	0.1
$\frac{dy}{dx}$	+ve	0	+ve
	/	—	/

There is a horizontal inflection point at $x = 0$.

As $x \to \infty$, the x^3 term in the function dominates so $y \to \infty$.
Similarly, as $x \to -\infty$, $y \to -\infty$.

The curve has a y-intercept of (0, 0).



Question 7

$$\frac{dy}{dx} = 4x^3 = 0$$

$$\Rightarrow x = 0$$

When $x = 0$,

$y = (0)^4 = 0$

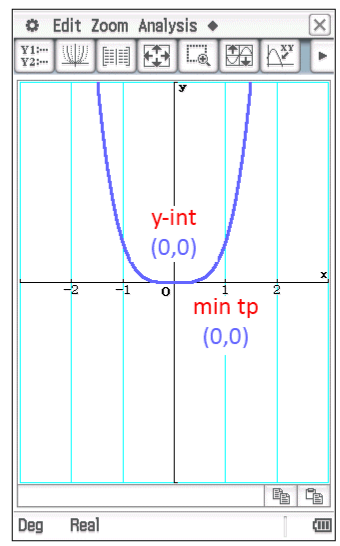
A stationary point exists at (0, 0).

Investigation of the sign of the derivative on either side will indicate the type of stationary point.

x	-0.1	0	0.1
$\frac{dy}{dx}$	$-$ve	0	$+$ve
	\	—	/

There is a minimum turning point at $x = 0$.

As $x \to \infty$, the x^4 term in the function dominates so $y \to \infty$.
Similarly, as $x \to -\infty$, $y \to \infty$.



Question 8

$$\frac{dy}{dx} = 6x - 3x^2 = 0$$

$$3x(2 - x) = 0$$

$$\Rightarrow x = 0, x = 2$$

When $x = 0$,

$y = 3(0)^2 - (0)^3 = 0$

When $x = 2$,

$y = 3(2)^2 - (2)^3 = 4$

Stationary points exist at (0, 0) and (2, 4).

Investigation of the sign of the derivative on either side will indicate the type of stationary point.

x	-0.1	0	0.1
$\frac{dy}{dx}$	$-$ve	0	$+$ve
	\	—	/

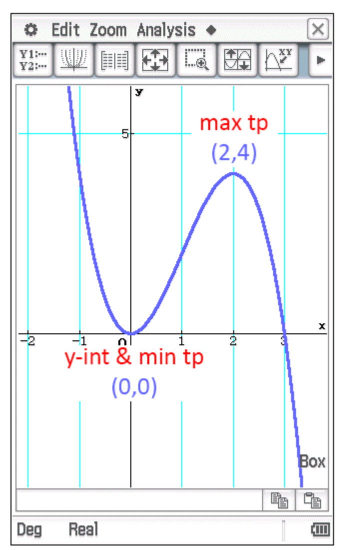
There is a minimum turning point at $x = 0$.

x	1.9	2	2.1
$\frac{dy}{dx}$	$+$ve	0	$-$ve
	/	—	\

There is a maximum turning point at $x = 2$.

As $x \to \infty$, the x^3 term in the function dominates so $y \to -\infty$.
Similarly, as $x \to -\infty$, $y \to \infty$.

The curve has a y-intercept of (0, 0).



Question 9

$$\frac{dy}{dx} = 4x - 4 = 0$$
$$\Rightarrow x = 1$$

When $x = 1$,
$y = 2(1)^2 - 4(1) + 7 = 5$

A stationary point exists at (1, 5).

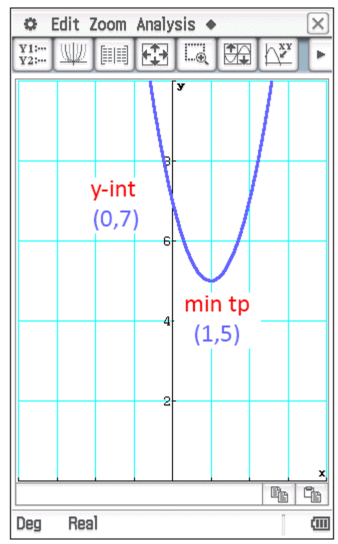
Investigation of the sign of the derivative on either side will indicate the type of stationary point.

x	0.9	1	1.1
$\frac{dy}{dx}$	$-$ve	0	$+$ve
	\	—	/

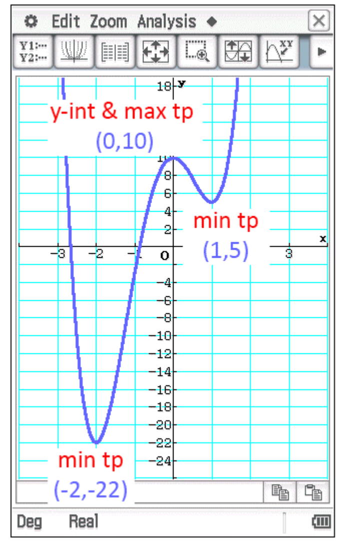
There is a minimum turning point at $x = 1$.

As $x \to \infty$, the $2x^2$ term in the function dominates so $y \to \infty$.
Similarly, as $x \to -\infty$, $y \to \infty$.

The curve has a y-intercept of (0, 7).



Question 10



$$\frac{dy}{dx} = 12x^3 + 12x^2 - 24x = 0$$

$$12x(x^2 + x - 2) = 0$$

$$12x(x-1)(x+2) = 0$$

$$\Rightarrow x = -2, x = 0, x = 1$$

When $x = -2$,

$y = 3(-2)^4 + 4(-2)^3 - 12(-2)^2 + 10 = -22$

When $x = 0$,

$y = 3(0)^4 + 4(0)^3 - 12(0)^2 + 10 = 10$

When $x = 1$,

$y = 3(1)^4 + 4(1)^3 - 12(1)^2 + 10 = 5$

Stationary points exist at (–2, –22), (0, 10) and (1, 5).

Investigation of the sign of the derivative on either side will indicate the type of stationary point.

x	-2.1	-2	-1.9
$\frac{dy}{dx}$	$-$ve	0	$+$ve
	\	—	/

There is a minimum turning point at $x = -2$.

x	–0.1	0	0.1
$\frac{dy}{dx}$	$+$ve	0	$-$ve
	/	—	\

There is a maximum turning point at $x = 0$.

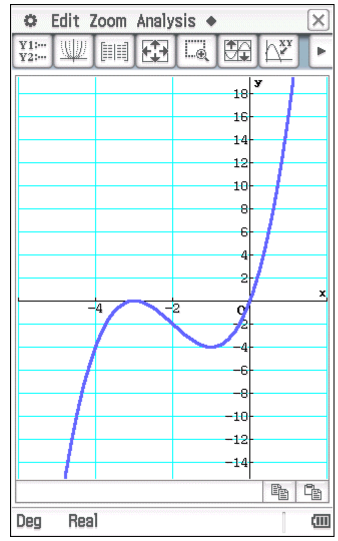
x	0.9	1	1.1
$\frac{dy}{dx}$	$-$ve	0	$+$ve
	\	—	/

There is a minimum turning point at $x = 1$.

As $x \to \infty$, the x^4 term in the function dominates so $y \to \infty$.
Similarly, as $x \to -\infty$, $y \to \infty$.

The curve has a y-intercept of (0, 10).

Question 11



a y-int, $x = 0$

$y = 0^3 + 6(0)^2 + 9(0) = 0$ (0, 0)

b x-int, $y = 0$

$$x^3 + 6x^2 + 9x = 0$$
$$x(x^2 + 6x + 9) = 0$$
$$x(x+3)^2 = 0$$
$$x = 0, x = -3$$

$(0, 0)$ and $(-3, 0)$

c As $x \to \infty, y \to \infty$. As $x \to -\infty, y \to -\infty$.

d $\frac{dy}{dx} = 3x^2 + 12x + 9 = 0$

$$3(x^2 + 4x + 3) = 0$$
$$3(x+3)(x+1) = 0$$
$$x = -3, x = -1$$

When $x = -3,\ y = (-3)^3 + 6(-3)^2 + 9(-3) = 0$

When $x = -1,\ y = (-1)^3 + 6(-1)^2 + 9(-1) = -4$

Stationary points exist at (–3, 0) and (–1, –4).

Investigation of the sign of the derivative on either side will indicate the type of stationary point.

x	-3.1	-3	-2.9
$\frac{dy}{dx}$	+ve	0	−ve
	/	—	\

There is a maximum turning point at $x = -3$.

x	-1.1	-1	-0.9
$\frac{dy}{dx}$	$-ve$	0	$+ve$
	\	—	/

There is a minimum turning point at $x = -1$.

e Minimum value of y in the required interval is –20 when x = –5.

Maximum value of y in the required interval is 16 when x= 1.

Question 12

$f'(x) = 6x^2 - 6x = 0$

$6x(x-1) = 0$

$x = 0, x = 1$

$f(0) = 2(0)^3 - 3(0)^2 = 0$

$f(1) = 2(1)^3 - 3(1)^2 = -1$

x	–0.1	0	0.1
$\frac{dy}{dx}$	+ve	0	−ve
	/	—	\

There is a maximum turning point at (0, 0).

x	0.9	1	1.1
$\frac{dy}{dx}$	−ve	0	+ve
	\	—	/

There is a minimum turning point at (1, –1).

a The minimum value of $f(x)$ for $x \ge 0$ occurs at the minimum turning point (1, –1).

The minimum value of $f(x)$ is –1.

b As (0, 0) is a maximum turning point, we need to find $f(-1)$.

$f(-1) = 2(-1)^3 - 3(-1)^2 = -5$ which is lower than the minimum turning point.

The minimum value of $f(x)$ for $-1 \le x \le 5$ is –5.

Exercise 6C

Question 1

$$\frac{dX}{dt} = 3t^2 - 30t + 48 = 0$$
$$3(t^2 - 10t - 16) = 0$$
$$3(t-8)(t-2) = 0$$
$$t = 2,\ 8$$

t	1.9	2	2.1
$\frac{dX}{dt}$	+ve	0	−ve
	/	—	\

There is a maximum turning point at $x = 2$.

t	7.9	8	8.1
$\frac{dX}{dt}$	−ve	0	+ve
	\	—	/

There is a minimum turning point at $x = 8$.

at $t = 8$
$$X = 8^3 - 15(8)^2 + 48(8) + 80$$
$$= 16$$

The maximum value of X is 16 which occurs when $t = 8$.

Question 2

$$\frac{dA}{dp} = 60 + 24p - 3p^2 = 0$$

$$-3(p^2 - 8p - 20) = 0$$

$$-3(p - 10)(p + 2) = 0$$

$$p = -2, 10$$

p	-2.1	-2	-1.9
$\frac{dA}{dp}$	$-$ve	0	$+$ve
	\	—	/

There is a minimum turning point at $x = -2$.

p	9.9	10	10.1
$\frac{dA}{dp}$	$+$ve	0	$-$ve
	/	—	\

There is a maximum turning point at $x = 10$.

at $p = 10$

$$A = 60(10) + 12(10)^2 - (10)^3 + 500$$

$$= 300$$

The maximum value of A is 300 which occurs when $p = 10$.

Question 3

$x = 20 + 5y$

$A = (20 - 5y)y$

$\quad = 20y - 5y^2$

$$\frac{dA}{dy} = 20 - 10y = 0$$

$$10y = 20$$

$$y = 2$$

y	1.9	2	2.1
$\frac{dA}{dy}$	$+$ve	0	$-$ve
	/	—	\

There is a maximum turning point at $y = 2$.

When $y = 2,$

$x = 20 - 5(2)$

$\quad = 10$

$A = 2 \times 10$

$\quad 20$

The maximum value of A is 20 which occurs when $x = 10, y = 2$.

Question 4

$$2x+3y=18 \Rightarrow y=\frac{18-2x}{3}$$

$$\begin{aligned} A &= x\left(\frac{18-2x}{3}\right) \\ &= 6x-\frac{2}{3}x^2 \end{aligned}$$

$$\begin{aligned} \frac{dA}{dx} = 6-\frac{4}{3}x &= 0 \\ \frac{4}{3}x &= 6 \\ x &= 4.5 \end{aligned}$$

x	4.4	4.5	4.6
$\frac{dA}{dx}$	+ve	0	−ve
	/	—	\

There is a maximum turning point at $x = 4.5$.

at $x = 4.5$

$$y=\frac{18-2(4.5)}{3}=3$$

$$\begin{aligned} A &= 4.5\times 3 \\ &= 13.5 \end{aligned}$$

The maximum value of A is 13.5 which occurs when $x = 4.5, y = 3$.

Question 5

Profit = Revenue – cost

$P(x) = R(x) - C(x)$

$= x(95 - x) - (500 + 25x)$

$= 95x - x^2 - 500 - 25x$

$= -x^2 + 70x - 500$

$\frac{dP}{dx} = -2x + 70 = 0$ (Negative coefficient of x2 indicates stationary point to be a max)

$2x = 70$

$x = 35$

x	34.9	35	35.1
$\frac{dP}{dx}$	$+$ve	0	$-$ve
	/	—	\

There is a maximum turning point at $x = 35$.

at $x = 35$

$P = -(35)^2 + 70(35) - 500$

$= 725$

The maximum profit of $725 occurs when 35 items are produced.

Question 6

$$P(x) = R(x) - C(x)$$
$$= x(300 - x) - (5000 + 60x)$$
$$= 300x - x^2 - 5000 - 60x$$
$$= -x^2 + 240x - 5000$$

$$\frac{dP}{dx} = -2x + 240 = 0$$
$$2x = 240$$ (Negative coefficient of x2 indicates stationary point to be a max)
$$x = 120$$

x	119	120	121
$\frac{dP}{dx}$	+ve	0	−ve
	/	—	\

There is a maximum turning point at $x = 120$.

at $x = 120$

$$P = -(120)^2 + 240(120) - 5000$$
$$= 9400$$

The maximum profit of $9400 occurs when 120 items are produced.

Question 7

Let x and y represent the dimensions of the enclosure.

a

$$2x + 2y = 100$$
$$x + y = 50 \Rightarrow y = 50 - x$$
$$A = xy$$
$$= x(50 - x)$$
$$= x^2 - 50x$$

$$\frac{dA}{dx} = 50 - 2x = 0$$
$$2x = 50$$
$$x = 25$$

x	24	25	26
$\frac{dA}{dx}$	+ve	0	−ve
	/	—	\

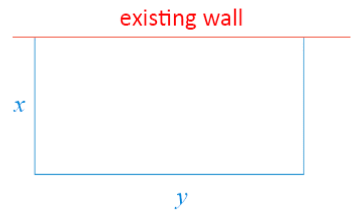
There is a maximum turning point at $x = 25$.

at $x = 25$

$$y = 50 - 25$$
$$= 25$$
$$A = 25 \times 25$$
$$625$$

The maximum area of 625 m^2 occurs when the length and width are both 25 m.

b $2x + y = 100 \Rightarrow y = 100 - 2x$



$$
\begin{aligned}
A &= xy \\
&= x(100 - 2x) \\
&= 100x - 2x^2
\end{aligned}
$$

$$
\begin{aligned}
\frac{dA}{dx} = 100 - 4x &= 0 \\
4x &= 100 \\
x &= 25
\end{aligned}
$$

x	24	25	26
$\frac{dA}{dx}$	+ve	0	−ve
	/	—	\

There is a maximum turning point at $x = 25$.

at $x = 25$

$$
\begin{aligned}
y &= 100 - 2(25) \\
&= 50 \\
A &= 25 \times 50 \\
&= 1250
\end{aligned}
$$

The maximum area of 1250 m^2 occurs when the length is 25 m and width is 50 m.

Question 8

$$\frac{dP}{dx} = 6000 - 200x = 0$$
$$200x = 6000$$
$$x = 30$$

x	29	30	31
$\frac{dP}{dx}$	+ve	0	−ve
	/	—	\

There is a maximum point at $x = 30$.

at $x = 30$

$$P = 50\ 000 + 6000(30) - 100(30)^2$$
$$= 140\ 000$$

The maximum profit of \$140 000 occurs when \$30 000 is spent on advertising.

Question 9

$4l + 4w + 4h = 6 \Rightarrow l + w + h = 1.5$

Given $l = 1.5w$

$1.5w + w + h = 1.5$

$2.5w + h = 1.5 \Rightarrow h = 1.5 - 2.5w$

Capacity is dependent on volume

$$\begin{aligned} V &= lwh \\ &= 1.5w \times w \times (1.5 - 2.5w) \\ &= 2.25w^2 - 3.75w^3 \end{aligned}$$

$$\frac{dV}{dw} = 4.5w - 11.25w^2 = 0$$

$$2.25w(2 - 5w) = 0$$

$$w = 0 \quad \text{or} \quad 2 - 5w = 0$$

$$w = 0.4$$

A width of zero makes no sense in this situation, so we will disregard it as a possible solution.

w	0.39	0.4	0.41
$\frac{dV}{dw}$	+ve	0	−ve
	/	—	\

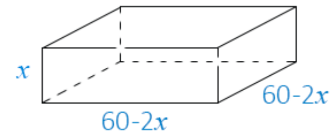
There is a maximum turning point at $w = 0.4$.

at $w = 0.4$

$$\begin{aligned} l &= 1.5 \times \times 0.4 \\ &= 0.6 \\ h &= 1.5 - 2.5(0.4) \\ &= 0.5 \\ V &= 0.4 \times 0.5 \times 0.6 \\ &= 0.12 \end{aligned}$$

The maximum capacity of 0.12 m^3 occurs with a width of 0.4 m, a length of 0.6 m and a height of 0.5 m.

Question 10



$$l = w = 60 - 2x$$

$$V = x(60 - 2x)^2$$

$$= 4x^3 - 240x^2 + 3600x$$

$$\frac{dV}{dx} = 12x^2 - 480x + 3600 = 0$$

$$12(x^2 + 40x + 300) = 0$$

$$12(x - 10)(x - 30) = 0$$

$$x = 10, x = 30$$

A square of size 30 cm would cut the card into 4 separate squares, so we will disregard $x = 30$ as a possible solution.

x	9.9	10	10.1
$\frac{dV}{dx}$	+ve	0	−ve
	/	—	\

There is a maximum turning point at $x = 10$.

at $x = 10$

$$V = 10[60 - 2(10)]^2$$

$$= 16\ 000$$

The maximum volume of 16 000 cm^3 occurs with when a square with 10 cm sides is removed.

Question 11

Let *x* represent the turn up, in cm

The open prism formed would have dimensions as follows:

$h = x,\ w = 24 - 2x$ and $l = 800$

$$\begin{aligned} V &= lwh \\ &= 800(24 - 2x)x \\ &= -1600x^2 + 19\ 200x \end{aligned}$$

$$\begin{aligned} \frac{dV}{dx} = -3200x + 19\ 200 &= 0 \\ 3200x &= 19\ 200 \\ x &= 6 \end{aligned}$$

(Negative coefficient of x^2 indicates stationary point to be a max.)

x	5.9	6	5.1
$\frac{dV}{dx}$	+ve	0	−ve
	/	—	\

There is a maximum point at $x = 6$.

at $x = 6$

$$\begin{aligned} V &= 800[24 - 2(6)] \times 6 \\ &= 57\ 600 \end{aligned}$$

The maximum volume of 57 600 cm^3 occurs with when 6 cm is turned up from each edge.

Question 12

a Let $C = 5000$

Let x be the number of 10c increases.

Tickets	Price	Revenue
7500	$1	7500 × 1
7250	$1.10	7250 × 1.1
7000	$1.20	7000 × 1.2

$R(x) = (7500 - 250x)(1 + 0.1x)$

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= (7500 - 250x)(1 + 0.1x) - 5000 \\ &= -25x^2 + 500x + 7500 - 5000 \\ &= -25x^2 + 500x + 2500 \end{aligned}$$

b $\frac{dP}{dx} = -50x + 500 = 0$ (Negative coefficient of x^2 indicates stationary point to be a max.)

$x = 10$

x	9.9	10	10.1
$\frac{dP}{dx}$	+ve	0	−ve
	/	—	\

There is a maximum point at $x = 10$.

c $1 + 0.1(10) = \$2$

d $7500 - 250(10) = 5000$ tickets

e $$\begin{aligned} P(10) &= -25(10)^2 + 500(10) + 2500 \\ &= \$5000 \end{aligned}$$

Question 13

$$\frac{dN}{dt}=6t^2-114t+288=0$$
$$6(t^2-19t+48)=0$$
$$6(t-16)(t-3)=0$$
$$t=3,\ t=16$$

t	2.9	3	3.1
$\frac{dN}{dt}$	+ve	0	−ve
	/	—	\

There is a maximum turning point at $t=3$.

t	15.9	16	16.1
$\frac{dN}{dt}$	−ve	0	+ve
	\	—	/

There is a minimum turning point at $t=16$.

As there is a minimum turning point when $t=16$ we need to check if the value at $t=24$ exceeds the value at $t=3,$ which was a local maximum.

at $t=3$
$$N=2(3)^3-57(3)^2+288(3)+2900$$
$$=3305$$
at $t=24$
$$N=2(24)^3-57(24)^2+288(24)+2900$$
$$=4628$$
at $t=16$
$$N=2(16)^3-57(16)^2+288(16)+2900$$
$$=1108$$

The maximum value of N is 4600 at $t=24,$ and the minimum value is 1100 at $t=16$.

Question 14

a $s = \frac{3^3}{3} - 6(3)^2 + 50(3)$

$= 105 \text{ m}$

b $v = \frac{ds}{dt} = (t^2 - 12t + 50) \text{ m/s}$

c $v = 0^2 - 12(0) + 50$

$= 50 \text{ m/s}$

d $\frac{dv}{dt} = 2t - 12 = 0$

$t = 6$

t	5.9	6	6.1
$\frac{dN}{dt}$	−ve	0	+ve
	\	_	/

There is a minimum turning point at $t = 6$.

$s = \frac{6^3}{3} - 6(6)^2 + 50(6)$

$= 156 \text{ m}$

Minimum velocity at $t = 6$ when the body is 156 m to the right of the origin.

Question 15

$\frac{dy}{dx} = -\frac{3x^2 + 140x + 1000}{50000} = 0$

$3x^2 + 140x + 1000 = 0$

$x = 8.8037, 37.8630$

As $0 \le x \le 20,$ we will disregard 37.8630 as a possible solution.
To the nearest centimetre, the sag occurs 880 cm from the origin.

at $x = 8.80, y = 0.0812$
$-0.0812 \text{ m} = 81.2 \text{ mm}$
the maximum sag is 81 mm

Question 16

Let x represent the number of \$500 increases

$$R = 100 + 5x$$
$$C = 100 - 2x$$
$$Z = (100 + 5x)(100 - 2x)$$
$$= -10x^2 + 300x + 10000$$

$$\frac{dZ}{dx} = -20x + 300 = 0$$
$$x = 15$$

(Negative coefficient of x^2 indicates stationary point to be a max.)

x	14.9	15	15.1
$\frac{dZ}{dx}$	+ve	0	−ve
	/	—	\

There is a maximum turning point at $x = 15$.

For a maximum Z score, the owner needs to spend \$12 500. $(5000 + 500 \times 15)$.

Exercise 6D

Question 1

$$P = 3r^2 + 5r^{-1}$$

$$\frac{dP}{dr} = 6r + (-1)5r^{-2}$$

$$= 6r - \frac{5}{r^2}$$

Question 2

$$A = 400r^{\frac{1}{2}}$$

$$\frac{dA}{dr} = (\frac{1}{2})400r^{-\frac{1}{2}}$$

$$= \frac{200}{\sqrt{r}}$$

a $\frac{dA}{dr} = \frac{200}{\sqrt{4}} = 100$

b $\frac{dA}{dr} = \frac{200}{\sqrt{25}} = 40$

c $\frac{dA}{dr} = \frac{200}{\sqrt{100}} = 20$

Question 3

$$\frac{dy}{dx} = 1 - \frac{2}{x^2} = 0$$
$$\frac{2}{x^2} = 1$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

at $x = \sqrt{2}$

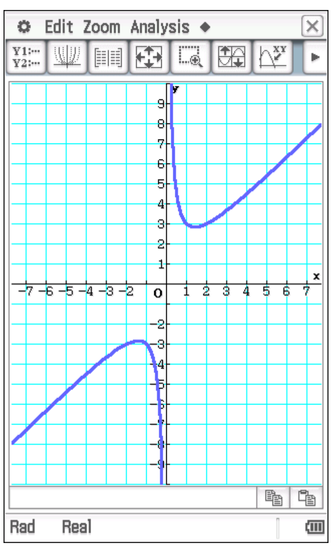
$$y = \sqrt{2} + \frac{2}{\sqrt{2}}$$
$$= \sqrt{2} + \sqrt{2}$$
$$= 2\sqrt{2}$$

at $x = -\sqrt{2}$

$$y = -\sqrt{2} + \frac{2}{(-\sqrt{2})}$$
$$= -2\sqrt{2}$$

Stationary points exist at $(\sqrt{2}, 2\sqrt{2})$ and $(-\sqrt{2}, -2\sqrt{2})$

From classpad display we can see $(\sqrt{2}, 2\sqrt{2})$ is a minimum point and $(-\sqrt{2}, -2\sqrt{2})$ is a maximum point.



Question 4

$$\frac{dy}{dx}=\frac{4}{x^2}-1=0$$
$$\frac{4}{x^2}=1$$
$$x^2=4$$
$$x=\pm 2$$

at $x=2$

$$y=5-\frac{4}{2}-2$$
$$=1$$

at $x=-2$

$$y=5-\frac{4}{(-2)}-(-2)$$
$$=9$$

Stationary points exist at $(-2, 9)$ and $(2, 1)$

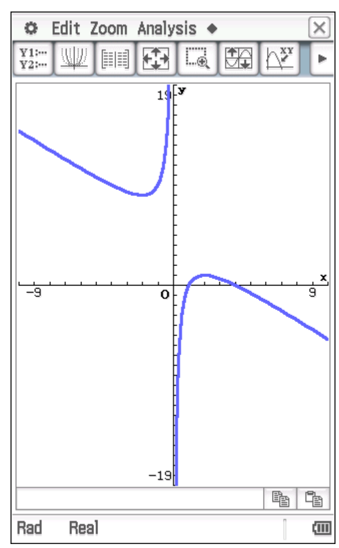
x	1.9	2	2.1
$\frac{dy}{dx}$	+ve	0	−ve
	/	—	\

There is a maximum turning point at $x=2$.

x	−2.1	−2	−1.9
$\frac{dy}{dx}$	−ve	0	+ve
	\	_	/

There is a minimum turning point at $x=-2$.

$(2, 1)$ is a maximum point and $(-2, 9)$ is a minimum point



Question 5

$$\frac{dy}{dx}=\frac{192}{x^3}+3=0$$

$$\frac{192}{x^3}=-3$$

$$x^3=-64$$

$$x=-4$$

at $x=-4$

$$y=3(-4)-\frac{96}{x^2}$$

$$=-18$$

A stationary point exists at $(-4,\,-18)$.

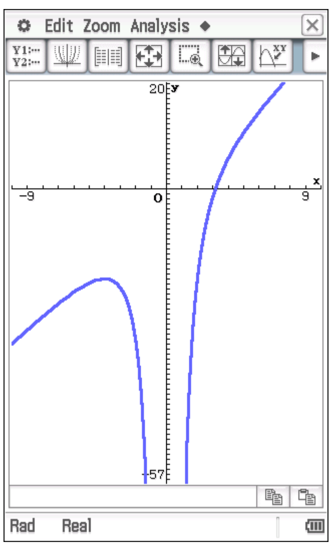
As $x\to 0, \frac{-96}{x^2}$ dominates the value and $y\to -\infty$.

This is true for postiive and negative values close to zero due to the squared x on the denominator.

As $x\to +\infty$, $3x$ dominates and $y\to +\infty$.

As $x\to -\infty$, $3x$ dominates and $y\to -\infty$.

For $x<0$, $(-4,\,-18)$ is a maximum point.



Question 6

a $V = x^2 y = 500 \Rightarrow y = \dfrac{500}{x^2}$

b $$\begin{aligned} A &= x^2 + 4xy \\ &= x^2 + 4x\left(\frac{500}{x^2}\right) \\ &= \left(x^2 + \frac{2000}{x}\right)\text{cm}^2 \end{aligned}$$

c $\dfrac{dA}{dx} = 2x - \dfrac{2000}{x^2} = 0$ (Positive coefficient of x^2 in A indicates this is a min point.)

$$\begin{aligned} 2x &= \frac{2000}{x^2} \\ 2x^3 &= 2000 \\ x^3 &= 1000 \\ x &= 10 \end{aligned}$$

x	9.9	10	10.1
$\dfrac{dA}{dx}$	$-$ve	0	$+$ve
	\	—	/

There is a minimum turning point at $x = 10$.

at $x = 10$

$$\begin{aligned} y &= \frac{500}{10^2} = 5 \\ A &= 10^2 + 4(5)(10) \\ &= 300 \end{aligned}$$

The minimum card required is 300 cm^2 when $x = 10$ cm, $y = 5$ cm.

Question 7

$V = \pi r^2 h = 535 \Rightarrow h = \dfrac{535}{\pi r^2}$

$$\begin{aligned} SA &= 2\pi r^2 + 2\pi r h \\ &= 2\pi r^2 + 2\pi r\left(\frac{535}{\pi r^2}\right) \\ &= 2\pi r^2 + \frac{1070}{r} \end{aligned}$$

$$\begin{aligned} \frac{dSA}{dt} = 4\pi r - \frac{1070}{r^2} &= 0 \\ 4\pi r &= \frac{1070}{r^2} \\ r^3 &= \frac{1070}{4\pi} \\ r &= 4.4 \end{aligned}$$

(Positive coefficient of r^2 in SA indicates this is a min point.)

r	4.3	4.4	4.5
$\dfrac{dSA}{dr}$	−ve	0	+ve
	\	—	/

There is a minimum turning point at $r = 4.4$.

at $r = 4.4$

$$\begin{aligned} h &= \frac{535}{\pi(4.4)^2} \\ &= 8.8 \text{ cm} \\ SA &= 2\pi(4.4)(4.4 + 8.8) \\ &= 365 \text{ cm}^2 \end{aligned}$$

Minimum surface area of 365 cm^2 when r = 4.4 cm and h = 8.8 cm.

Question 8

$V = \pi r^2 h = 535 \Rightarrow h = \dfrac{535}{\pi r^2}$

If the base costs twice as much, for any arbitrary cost per square cm,

$$\begin{aligned} C &= 2\pi r^2 \times 2 + 2\pi r h \\ &= 4\pi r^2 + 2\pi r\left(\frac{535}{\pi r^2}\right) \\ &= 4\pi r^2 + \frac{1070}{r} \end{aligned}$$

$$\frac{dC}{dr} = 8\pi r - \frac{1070}{r^2} = 0$$

(Positive coefficient of x2 in C indicates this is a min point.)

$$\begin{aligned} 8\pi r &= \frac{1070}{r^2} \\ r^3 &= \frac{1070}{8\pi} \\ &= 3.492 \\ r &= 3.5 \text{ (to 1 dp)} \end{aligned}$$

r	3.4	3.5	3.6
$\dfrac{dC}{dr}$	$-$ve	0	$+$ve
	\	__	/

There is a minimum turning point at $r = 3.5$.

at $r = 3.5$

$$\begin{aligned} h &= \frac{535}{\pi(3.492)^2} \\ &= 13.96 \\ h &= 14.0 \text{ (to 1dp)} \end{aligned}$$

The cost is minimised when the radius is 3.5 cm and the height is 13.9 cm.

Question 1

a 5^2

b 5^4

c 5^3

d 5^0

e 5^3

f 5^6

g 5^5

h 5^4

i $(5^3)^2 \times 5 = 5^7$

j 5^3

k 5^2

l 5^1

m 5^{10}

n 5^4

o 5^{17}

p 5^2

q 5^6

r 5^3

s 5^5

t 5^6

u $\dfrac{5^8 \times 5^2}{5^3} = 5^7$

v 5^8

w 5^2

x $\frac{5^8}{5^3 \times 5^2} = 5^3$

y $3^2 + 4^2 = 25$

$25 = 5^2$

z $\frac{36+14}{2} = 25$

$25 = 5^2$

Question 2

a $\frac{(a^3 \times a^{\frac{1}{2}})^2}{a^3}$

$= \frac{a^7}{a^3}$

$= a^4$

b $\frac{5^3 b^{-6} a^3}{5^2 a^{-4} b^2}$

$= \frac{5a^3 a^4}{b^2 b^6}$

$= \frac{5a^7}{b^8}$

c $\frac{2^n \times 2^1 + 2^n \times 2^n}{2^n}$

$= \frac{2^n(2+2^n)}{2^n}$

$= 2 + 2^n$

d $\frac{5x^3(x+2x^4)}{5x^3}$

$= x + 2x^4$

e $\dfrac{2^x + 2^x \times 2^3}{3^2}$

$= \dfrac{2^x(1+2^3)}{3^2}$

$= 2^x$

f $\dfrac{3^n \times 3 - 15}{5 \times 3^n - 25}$

$= \dfrac{3(3^n - 5)}{5(3^n - 5)}$

$= \dfrac{3}{5}$

Question 3

$T_4 = a + 3d = 130$

$d = 11$

$a + 3(11) = 130$

$a = 97$

The first six terms are 97, 108, 119, 130, 141, 152.

Question 4

$T_4 = ar^3 = 2.8$

$r = 0.2$

$a(0.2)^3 = 2.8$

$a = 350$

The first five terms are 350, 70, 14, 2.8, 0.56.

Question 5

a $\frac{18-0}{6-3}=6$

b $\frac{dy}{dx}=2x-3$

at $x=3$

$$\frac{dy}{dx}==2(3)-3$$
$$=3$$

c at $x=3$

$$\frac{dy}{dx}=2(6)-3$$
$$=9$$

Question 6

a $\frac{198-18}{6-3}=60$

b $\frac{dy}{dx}=3x^2-3$

at $x=3$

$$\frac{dy}{dx}=3(3)^2-3$$
$$=24$$

c at $x=6$

$$\frac{dy}{dx}=3(6)^2-3$$
$$=105$$

Question 7

a 15 pills are shown in the diagram.

b **i** $1+2+3+\ldots \Rightarrow AP$ with $a=1, d=1$

$$S_{10}=\frac{10}{5}[2(1)+9(1)]$$
$$=55$$

ii $$S_{15}=\frac{15}{5}[2(1)+14(1)]$$
$$=120$$

Question 8

a $a=3, d=9$

$$507=3+(n-1)9$$
$$504=9(n-1)$$
$$56=n-1$$
$$n=57$$

$$S_n=\frac{57}{2}(3+507)$$
$$=14535$$

b $a=30, r=-3$

$$S_{10}=\frac{30[(-3)^{10}-1]}{-3-1}$$
$$=-442860$$

c $a=6, r=-2$

$$T_n=ar^{n-1}$$
$$6\ 291\ 456=6\times 2^{n-1}$$
$$1\ 048\ 576=2^{n-1}$$
$$n-1=20$$
$$n=21$$

$$S_{21}=\frac{6(2^{21}-1)}{2-1}$$
$$=12\ 582\ 906$$

d $a = 100, r = 0.8$

$$S_\infty = \frac{100}{1-0.8} = 500$$

e 5

Question 9

The tangent at $x=1$ passes through (0, –2) and (1, 1).

The gradient of this line, and hence the gradient of $f(x)$ at this point is 3.

Then tangent at $x=2$ passes through (0, –16) and (2, 8).

The gradient of this line, and hence the gradient of $f(x)$ at this point is $\frac{8-(-16)}{2}=12$.

Using calculus,

$$f(x) = x^3 \text{ and } f'(x) = 3x^2$$
$$f'(1) = 3(1)^2 = 3$$
$$f'(2) = 3(2)^2 = 12$$

Question 10

$$\frac{dy}{dx} = (x+4)(2x-3) = 0$$
$$x+4=0 \quad \text{or} \quad 2x+3=0$$
$$x=-4 \qquad x=-\frac{3}{2}$$

There are two places with a zero gradient.

Question 11

a

$$\lim_{h\to 0}\frac{f(5+h)-f(5)}{h}$$
$$=\lim_{h\to 0}\frac{(5+h)^2-5^2}{h}$$
$$=\lim_{h\to 0}\frac{25+10h+h^2-25}{h}$$
$$=\lim_{h\to 0}\frac{h^2+10h}{h}$$
$$=\lim_{h\to 0}\frac{h(h+10)}{h}$$
$$=\lim_{h\to 0} h+10$$
$$=10$$

b

$$\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$
$$=\lim_{h\to 0}\frac{[(1+h)^2+(1+h)]-[1^2+1]}{h}$$
$$=\lim_{h\to 0}\frac{h^2+2h+1+1+h-2}{h}$$
$$=\lim_{h\to 0}\frac{h^2+3h}{h}$$
$$=\lim_{h\to 0}\frac{h(h+3)}{h}$$
$$=\lim_{h\to 0} h+3$$
$$=3$$

c

$$\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$
$$=\lim_{h\to 0}\frac{[(2+h)^3+(2+h)]-[2^3+2]}{h}$$
$$=\lim_{h\to 0}\frac{h^3+6h^2+12h+8+2+h-10}{h}$$
$$=\lim_{h\to 0}\frac{h^3+6h^2+13h}{h}$$
$$=\lim_{h\to 0}\frac{h(h^2+6h+13)}{h}$$
$$=\lim_{h\to 0}h^2+6h+13$$
$$=13$$

Question 12

a D, H, K, P

b B, F, I, K, N, O

c G, H, L, M

d A, C, D, E, J, P

Question 13

$$y=7x-3\Rightarrow m=7$$
$$\frac{dy}{dx}=2x+5=7$$
$$2x=2$$
$$x=1$$
at $x=1$
$$y=(1)^2+5(1)-4$$
$$=2$$

at the point (1, 2)

Question 14

a $\frac{dy}{dx} = 2 - x^{-2}$

$= 21 - \frac{1}{x^2}$

$1 = 2 - \frac{1}{x^2}$

$\frac{1}{x^2} = 1$

$x^2 = 1$

$x = \pm 1$

at $x = 1$,

$y = 2(1) + \frac{1}{1}$

$= 3$

at $x = -1$

$y = 2(-1) + \frac{1}{(-1)}$

$= -3$

at (1, 3) and (−1, −3)

b $\frac{dy}{dx} = 3 - \frac{1}{2}.4x^{-\frac{1}{2}}$

$-1 = 3 - \frac{2}{\sqrt{x}}$

$\frac{2}{\sqrt{x}} = 4$

$\sqrt{x} = \frac{1}{2}$

$x = \frac{1}{4}$

at $x = \frac{1}{4}$

$y = 3\left(\frac{1}{4}\right) - 4\sqrt{\frac{1}{4}}$

$= -1.25$

at (0.25, −1.25)

Question 15

a $f(21) = 2(21)^4 - 5(21)^3 + (21)^2 - 2(21) + 6$

$= 343\,062$

$f(31) = 2(31)^4 - 5(31)^3 + (31)^2 - 2(31) + 6$

$= 1\,698\,992$

$f(41) = 2(41)^4 - 5(41)^3 + (41)^2 - 2(41) + 6$

$= 5\,308\,522$

b $f'(x) = 8x^3 - 15x^2 + 2x - 2$

$f'(21) = 8(21)^3 - 15(21)^2 + 2(21) - 2$

$= 67\,513$

$f'(31) = 8(31)^3 - 15(31)^2 + 2(31) - 2$

$= 223\,973$

$f'(x) = 8(41)^3 - 15(41)^2 + 2(41) - 2$

$= 526\,233$

Question 16

$$\frac{dy}{dx} = 3x^2 + 6x - 20 = 25$$

$$3x^2 + 6x - 45 = 0$$

$$3(x^2 - 2x + 15) = 0$$

$$2(x - 5)(x + 3) = 0$$

$$x = 3, x = -5$$

at $x = -5$

$$y = (-5)^3 + 3(-5)^2 - 20(-5) + 10$$

$$= 60$$

at $x = 3$

$$y = (3)^3 + 3(3)^2 - 20(3) + 10$$

$$= 4$$

at $x = -5$

$$\frac{dy}{dx} = 3(-5)^2 + 6(-5) - 20$$

$$= 25$$

$$y = 25x + c$$

$$60 = 25(-5) + c$$

$$c = 185$$

Equation of tangent at $(-5, 60)$ is $y = 25x + 185$.

at $x = 3$

$$\frac{dy}{dx} = 3(3)^2 + 6(3) - 20$$

$$= 25$$

$$y = 25x + c$$

$$4 = 25(3) + c$$

$$c = -71$$

Equation of tangent at (3, 4) is $y = 25x - 71$.

Question 17

a

p	9.5	10.5
N	70	50

$$a = m = -20$$
$$N = -20p + c$$
$$70 = -20(9.5) + c$$
$$c = 260$$
$$\therefore N = 260 - 20p$$

b

$$R = p \times N$$
$$= p(260 - 20p)$$
$$= 260p - p^2$$

c Profit = Revenue – cost

Revenue from selling N kg: $260p - p^2$

Cost pf purchasing N kg: $7N = 7(260 - 20p)$

$$\text{Profit} = 260p - 20p^2 - 7(260 - 20p)$$
$$= -20p^2 + 400p - 1820$$

d

$$\frac{dP}{dp} = -40p + 400 = 0$$
$$p = 10$$

p	9.9	10	10.1
$\frac{dP}{dp}$	+ve	0	−ve
	/	—	\

There is a maximum point at $p = 10$.

$$P = -20(10)^2 + 400(10) - 1820$$
$$= 180$$

A maximum profit of $180 is achieved when 60 kg are sold at $10/kg.

Question 18

a $7\times10^{12}\times2\times10^{11}$

$=14\times10^{23}$

$=1.4\times10^{24}$

b $7\times10^{12}+2\times10^{11}$

$=70\times10^{11}+2\times10^{11}$

$=72\times10^{11}$

$=7.2\times10^{12}$

c $7\times10^{12}-2\times10^{11}$

$=7\times10^{12}-0.2\times10^{12}$

$=6.8\times10^{12}$

d $\dfrac{7\times10^{12}}{2\times10^{11}}$

$=3.5\times10^{1}$

e $5\times7\times10^{12}\times2\times10^{11}$

$=70\times10^{23}$

$=7\times10^{24}$

f $\dfrac{\left(7\times10^{12}\right)^2}{2\times10^{11}}$

$=\dfrac{49\times10^{24}}{2\times10^{11}}$

$=24.5\times10^{13}$

$=2.45\times10^{14}$

Question 19

a $N = (0)^3 + 30(0) + 200$

$= 200$

b $N = (10)^3 + 30(10) + 200$

$= 1500$

c $\dfrac{1500 - 200}{10} = 130$ organisms/h

d **i** $\dfrac{dN}{dt} = 3t^2 + 30$

at $t = 0$

$\dfrac{dN}{dt} = 3(0)^2 + 30$

$= 30$

ii at $t = 5$

$\dfrac{dN}{dt} = 3(5)^2 + 30$

$= 105$

iii at $t = 10$

$\dfrac{dN}{dt} = 3(10)^2 + 30$

$= 330$

Question 20

a The function has an average rate of change of 41 units per unit of x between $x = 1$ and $x = 3$.

b The function has an instantaneous rate of change of 109 at $x = 3$.

Question 21

a

C	10	9.80	9.60
S	500	525	550

$C = (10 - 0.2x)$ / m

b $S = 500 + 25x$

c
$$\begin{aligned} R &= C \times S \\ &= (10 - 0.2x)(500 + 25x) \\ &= -5x^2 + 150x + 5000 \end{aligned}$$

d
$$\frac{dR}{dx} = -10x + 150 = 0$$
$$x = 15$$

x	14.9	15	15.1
$\frac{dR}{dx}$	+ve	0	−ve
	/	—	\

There is a maximum point at $x = 15$.

$$\begin{aligned} R &= -5(15)^2 + 150(15) + 5000 \\ &= 6125 \end{aligned}$$

Maximum revenue of $6125 when the price is reduced by 20 × 15 times, i.e. a reduction of $3.